

### III. Electronic Noise

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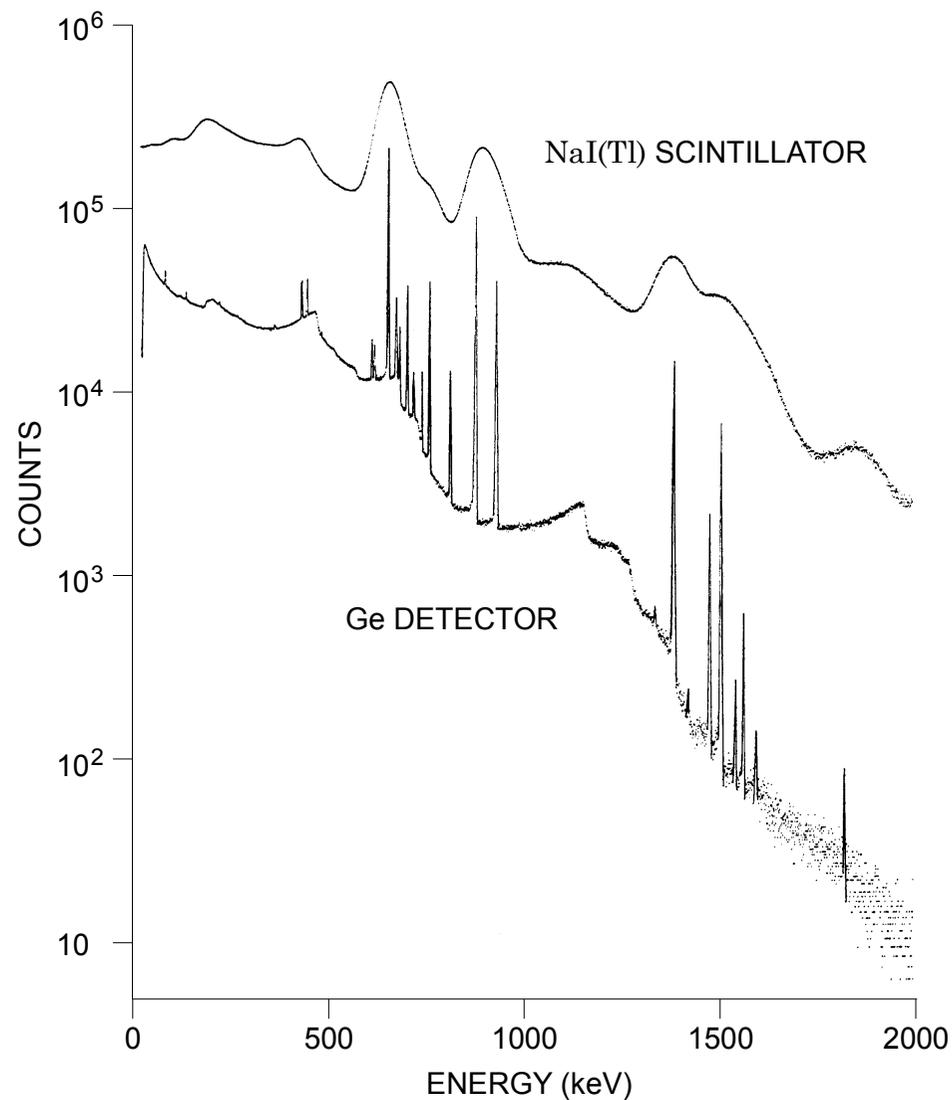
## 1. Resolution and Electronic Noise

Resolution: the ability to distinguish signal levels

### 1. Why?

a) Recognize structure in amplitude spectra

Comparison between NaI(Tl) and Ge detectors

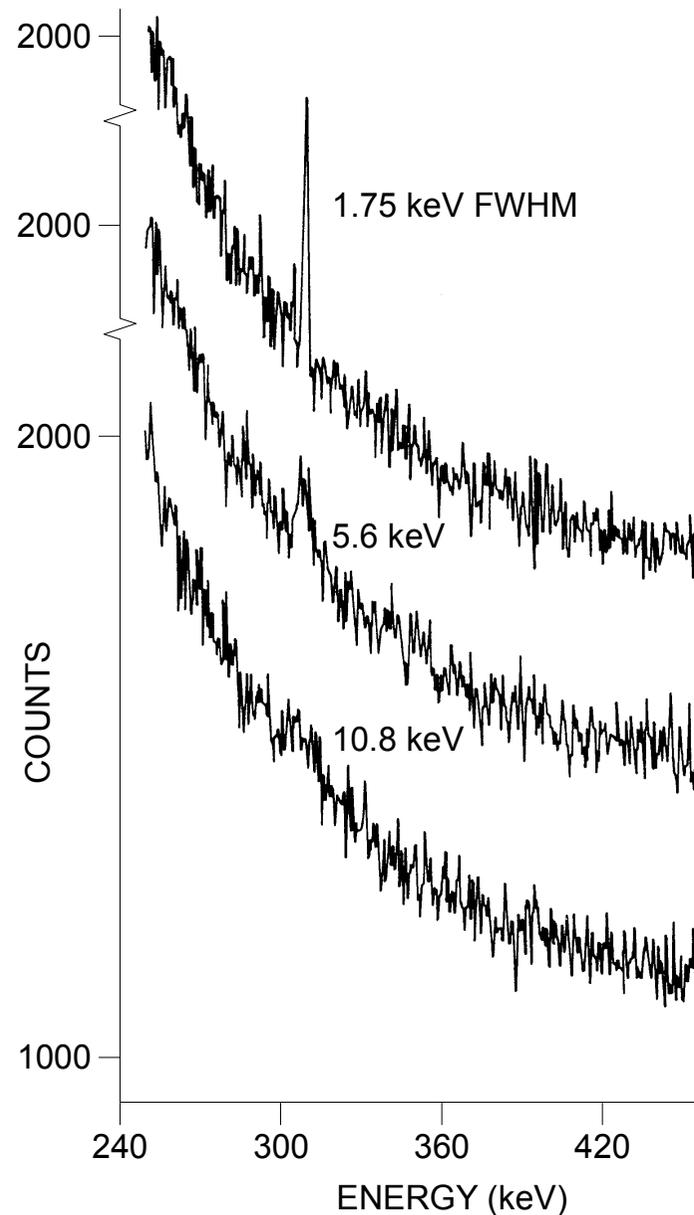


J.Cl. Philippot, IEEE Trans. Nucl. Sci. **NS-17/3** (1970) 446

## b) Improve sensitivity

Signal to background ratio improves with better resolution

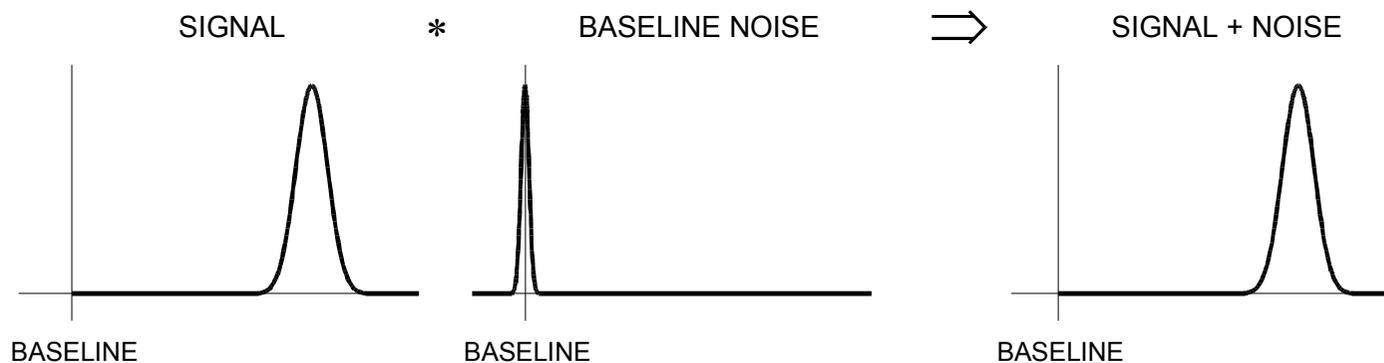
(signal counts in fewer bins compete with fewer background counts)



G.A. Armantrout *et al.*, IEEE Trans. Nucl. Sci. **NS-19/1** (1972) 107

## What determines Resolution?

### a) Signal Variance $\gg$ Baseline Variance



$\Rightarrow$  Electronic (baseline) noise not important

- Examples:
- High-gain proportional chambers
  - Scintillation Counters with High-Gain PMTs

1 MeV  $\gamma$ -rays absorbed by NaI(Tl) crystal:

$$\text{Number of photoelectrons: } N_{pe} \approx 8 \cdot 10^4 [\text{MeV}^{-1}] \times E_\gamma \times QE \approx 2.4 \cdot 10^4$$

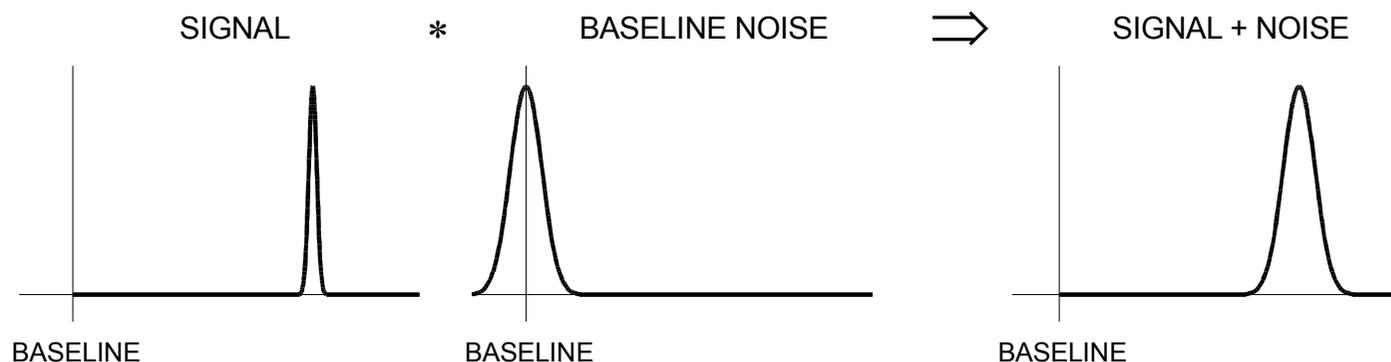
$$\text{Variance typically: } \sigma_{pe} = N_{pe}^{1/2} \approx 160 \text{ and } \sigma_{pe} / N_{pe} \approx 5 - 8\%$$

$$\text{Signal at PMT anode (Gain= } 10^4\text{): } Q_{sig} = G_{PMT} N_{pe} \approx 2.4 \cdot 10^8 \text{ el}$$

$$\text{and variance } \sigma_{sig} = G_{PMT} \sigma_{pe} \approx 1.2 \cdot 10^7 \text{ el}$$

whereas electronic noise easily  $< 10^4$  el

## b) Signal Variance $\ll$ Baseline Variance



⇒ Electronic (baseline) noise critical for resolution

- Examples:
- Gaseous ionization chambers (no internal gain)
  - Semiconductor detectors

e.g. in Si : Number of electron-hole pairs  $N = \frac{E_{dep}}{3.6 \text{ eV}}$

Variance  $\sigma = \sqrt{F \cdot N}$  (where  $F$  = Fano factor  $\approx 0.1$ )

For 50 keV photons:  $\sigma \approx 40 \text{ el} \Rightarrow \sigma / N = 7.5 \cdot 10^{-4}$

Obtainable noise levels are 10 to 1000 el.

Baseline fluctuations can have many origins ...

pickup of external interference

artifacts due to imperfect electronics

... etc.,

but the (practical) fundamental limit is electronic noise.

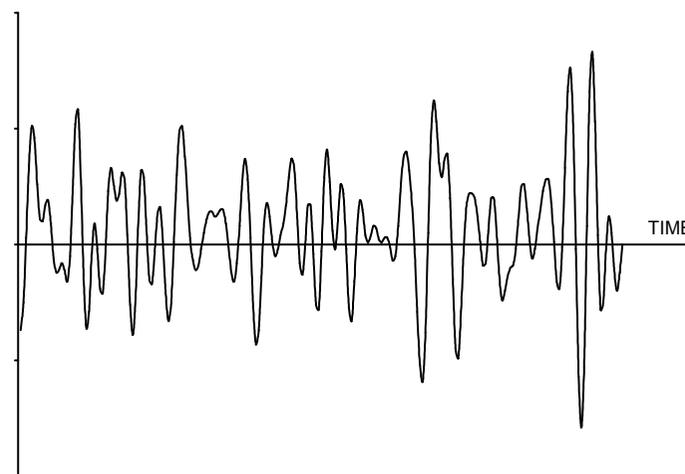
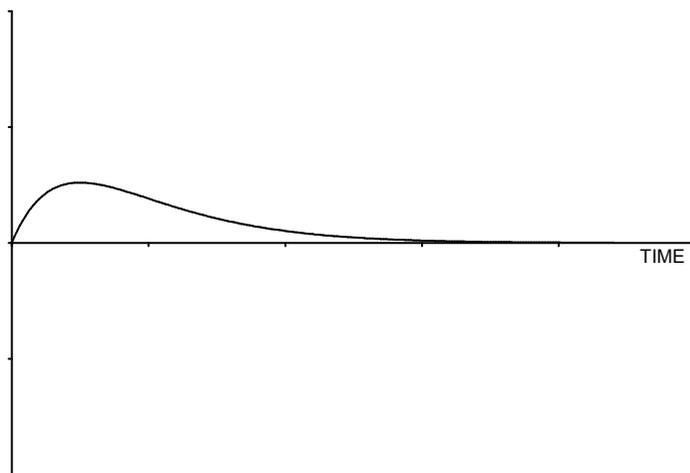
## Electronic Noise

Choose a time when no signal is present.

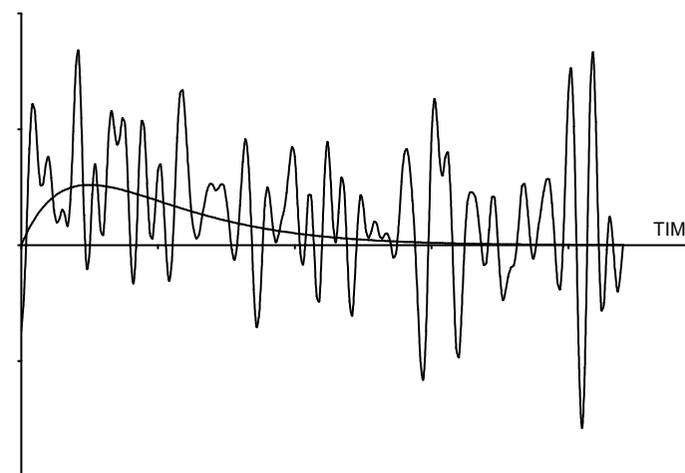
Amplifier's quiescent output level (baseline):

In the presence of a signal, noise + signal add.

Signal



Signal+Noise ( $S/N = 1$ )



$S/N \equiv$  peak signal to rms noise

Measurement of peak amplitude yields signal amplitude + noise fluctuation

The preceding example could imply that the fluctuations tend to increase the measured amplitude, since the noise fluctuations vary more rapidly than the signal.

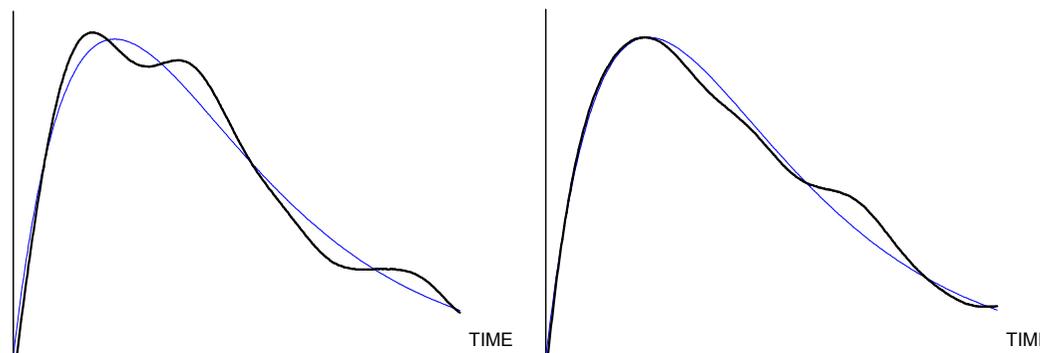
In an optimized system, the time scale of the fluctuation is comparable to the signal peaking time.

Then the measured amplitude fluctuates positive and negative relative to the ideal signal.

Measurements taken at 4  
different times:

noiseless signal superimposed  
for comparison

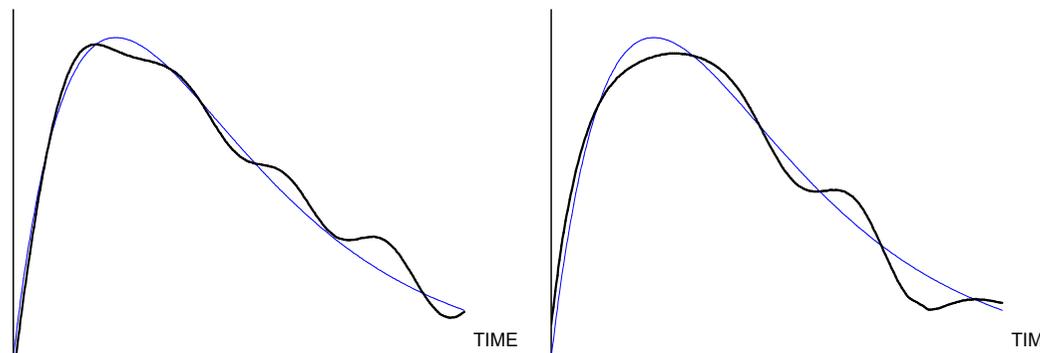
$$S/N = 20$$



Noise affects

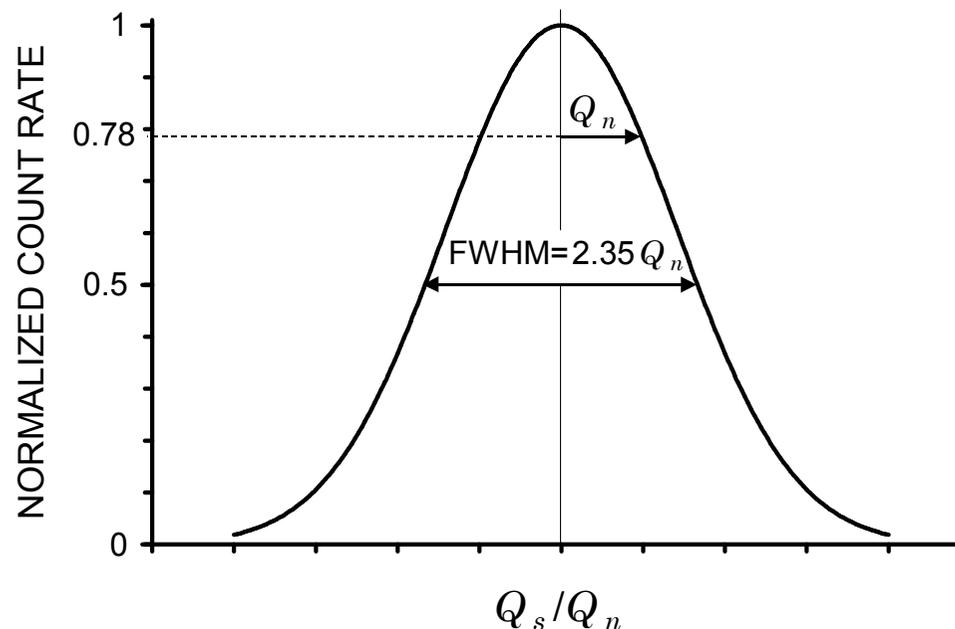
Peak signal

Time distribution



Electronic noise is purely random.

- ⇒ amplitude distribution is Gaussian
- ⇒ noise modulates baseline
- ⇒ baseline fluctuations superimposed on signal
- ⇒ output signal has Gaussian distribution



## Measuring Resolution

Inject an input signal with known charge using a pulse generator set to approximate the detector signal shape.

Measure the pulse height spectrum.

peak centroid ⇒ signal magnitude  
 peak width ⇒ noise (FWHM= 2.35  $Q_n$ )

## 2. Basic Noise Mechanisms and Characteristics

Consider  $n$  carriers of charge  $e$  moving with a velocity  $v$  through a sample of length  $l$ . The induced current  $i$  at the ends of the sample is

$$i = \frac{n e v}{l}$$

The fluctuation of this current is given by the total differential

$$\langle di \rangle^2 = \left( \frac{ne}{l} \langle dv \rangle \right)^2 + \left( \frac{ev}{l} \langle dn \rangle \right)^2,$$

where the two terms are added in quadrature since they are statistically uncorrelated.

Two mechanisms contribute to the total noise:

- velocity fluctuations, e.g. thermal noise
- number fluctuations, e.g. shot noise  
excess or “1/f” noise

Thermal noise and shot noise are both “white” noise sources, i.e.

power per unit bandwidth ( $\equiv$  spectral density) is constant:  $\frac{dP_{noise}}{df} = const.$

## Thermal Noise in Resistors

The most common example of noise due to velocity fluctuations is the thermal noise of resistors.

Noise power density vs. frequency  $f$ :  $\frac{dP_{noise}}{df} = 4kT$        $k = \text{Boltzmann constant}$

$T = \text{absolute temperature}$

since  $P = \frac{V^2}{R} = I^2 R$

$R = \text{DC resistance}$

the spectral noise voltage density  $\frac{dV_{noise}^2}{df} \equiv e_n^2 = 4kTR$

and the spectral noise current density  $\frac{dI_{noise}^2}{df} \equiv i_n^2 = \frac{4kT}{R}$

The total noise depends on the bandwidth of the system.

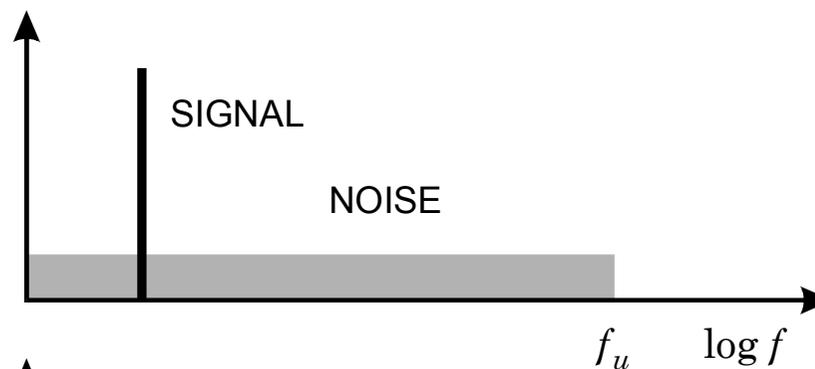
For example, the total noise voltage at the output of a voltage amplifier with the frequency dependent gain  $A_v(f)$  is

$$v_{on}^2 = \int_0^{\infty} e_n^2 A_v^2(f) df$$

Note: Since spectral noise components are not correlated, one must integrate over the noise power (proportional to voltage or current squared).

Total noise increases with bandwidth.

Total noise is the integral over the shaded region.



$S/N$  increases as noise bandwidth is reduced until signal components are attenuated significantly.



## Shot noise

A common example of noise due to number fluctuations is “shot noise”, which occurs whenever carriers are injected into a sample volume independently of one another.

Example: current flow in a semiconductor diode  
(emission over a barrier)

Spectral noise current density:  $i_n^2 = 2eI$        $e = \text{electronic charge}$   
 $I = \text{DC current}$

A more intuitive interpretation of this expression will be given later.

*Note:* Shot noise does not occur in “ohmic” conductors. Since the number of available charges is not limited, the fields caused by local fluctuations in the charge density draw in additional carriers to equalize the total number.

## Low Frequency (“1/f”) Noise

Charge can be trapped and then released after a characteristic lifetime  $\tau$ .

The spectral density for a single lifetime

$$S(f) \propto \frac{\tau}{1 + (2\pi f\tau)^2} .$$

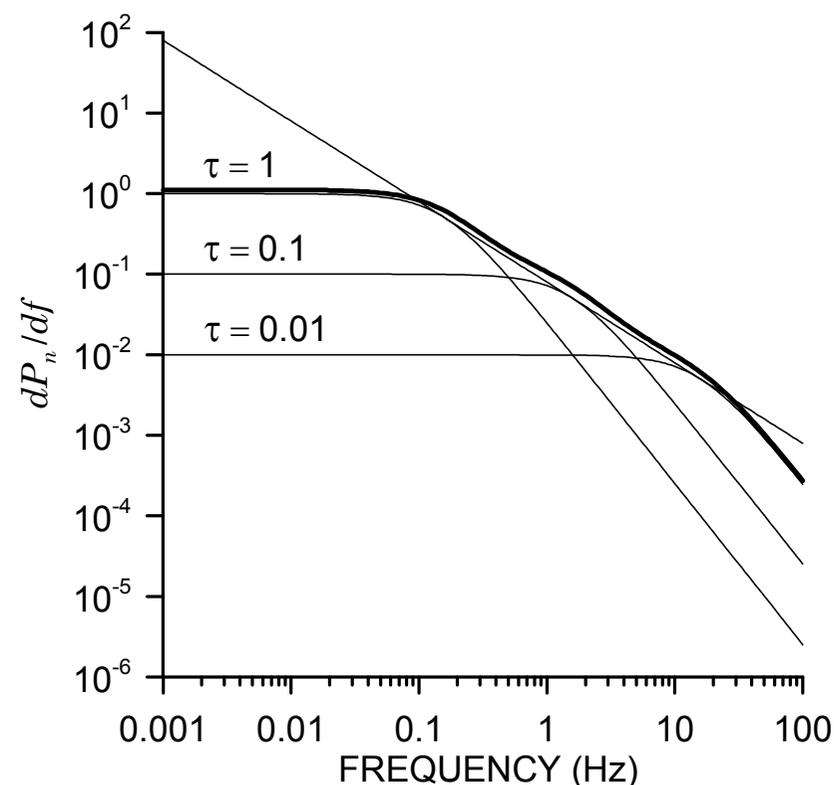
For  $2\pi f\tau \gg 1$ :  $S(f) \propto \frac{1}{f^2}$  .

However,  
several traps with different time constants  
can yield a “1/f” distribution:

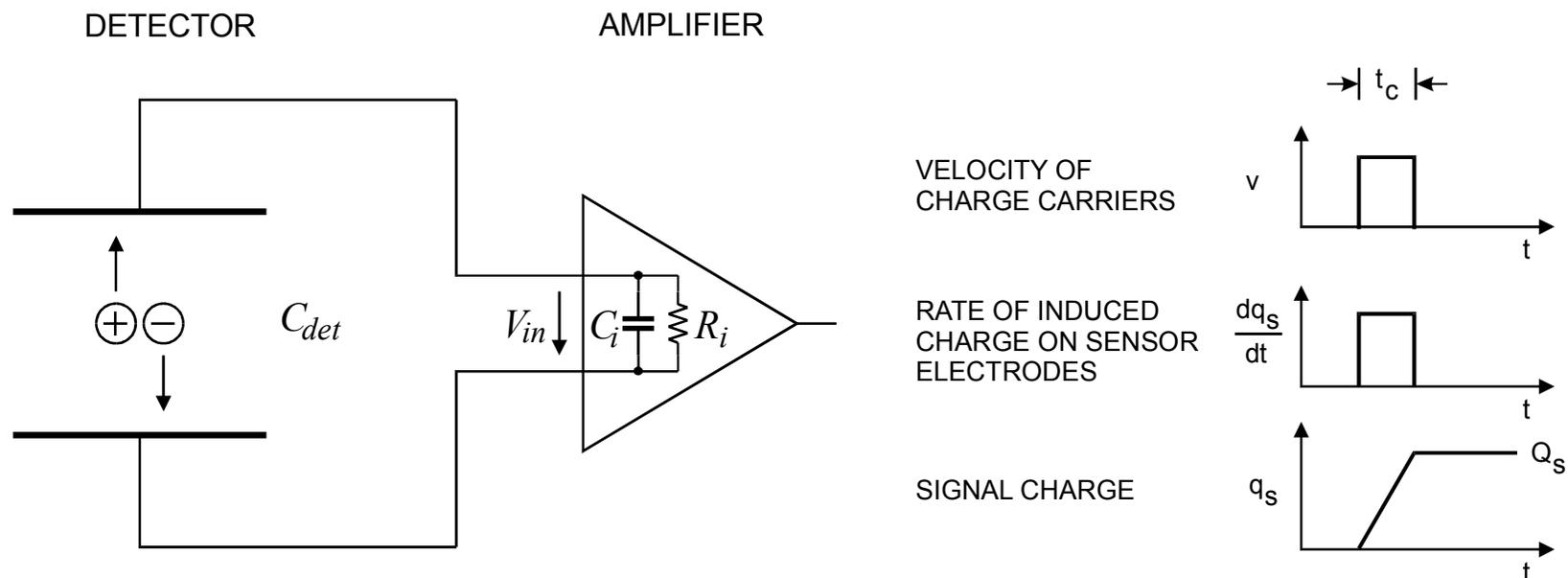
Traps with three time constants of  
0.01, 0.1 and 1 s yield a 1/f distribution  
over two decades in frequency.

Low frequency noise is ubiquitous – must  
not have 1/f dependence, but commonly  
called 1/f noise.

Spectral power density:  $\frac{dP_{noise}}{df} = \frac{1}{f^\alpha}$  (typically  $\alpha = 0.5 - 2$ )



### 3. Signal-to-Noise Ratio vs. Detector Capacitance



if  $R_i \times (C_{det} + C_i) \gg$  collection time,

$$\text{peak voltage at amplifier input } V_{in} = \frac{Q_s}{C} = \frac{\int i_s dt}{C} = \frac{Q_s}{C_{det} + C_i}$$

↑

Magnitude of voltage depends on total capacitance at input!

The peak amplifier signal  $V_S$  is inversely proportional to the **total capacitance at the input**, i.e. the sum of

1. detector capacitance,
2. input capacitance of the amplifier, and
3. stray capacitances.

Assume an amplifier with a noise voltage  $v_n$  at the input.

Then the signal-to-noise ratio

$$\frac{S}{N} = \frac{V_S}{v_n} \propto \frac{1}{C}$$

- However,  $S/N$  does not become infinite as  $C \rightarrow 0$   
(then front-end operates in current mode)
- The result that  $S/N \propto 1/C$  generally applies to systems that measure signal charge.

## Noise vs. Detector Capacitance – Charge-Sensitive Amplifier

In a voltage-sensitive preamplifier

- noise voltage at the output is essentially independent of detector capacitance,
- input signal decreases with increasing input capacitance, so signal-to-noise ratio depends on detector capacitance.

In a charge-sensitive preamplifier, the signal at the amplifier output is independent of detector capacitance (if  $C_i \gg C_d$ ).

What is the noise behavior?

- Noise appearing at the output of the preamplifier is fed back to the input, decreasing the output noise from the open-loop value  $v_{no} = v_{ni} A_v$ .
- The magnitude of the feedback depends on the shunt impedance at the input, i.e. the detector capacitance.

Although specified as an equivalent input noise, the dominant noise sources are typically internal to the amplifier.

Only in a fed-back configuration is some of this noise actually present at the input. In other words, the primary noise signal is not a physical charge (or voltage) at the amplifier input to which the loop responds in the same manner as to a detector signal.

**$\Rightarrow$   $S/N$  at the amplifier output depends on feedback.**

## Noise in charge-sensitive preamplifiers

Start with an output noise voltage  $v_{no}$ , which is fed back to the input through the capacitive voltage divider  $C_f - C_d$ .

$$v_{no} = v_{ni} \frac{X_{C_f} + X_{C_d}}{X_{C_d}} = v_{ni} \frac{\frac{1}{\omega C_f} + \frac{1}{\omega C_d}}{\frac{1}{\omega C_d}}$$

$$v_{no} = v_{ni} \left( 1 + \frac{C_d}{C_f} \right)$$

Equivalent input noise charge

$$Q_{ni} = \frac{v_{no}}{A_Q} = v_{no} C_f$$

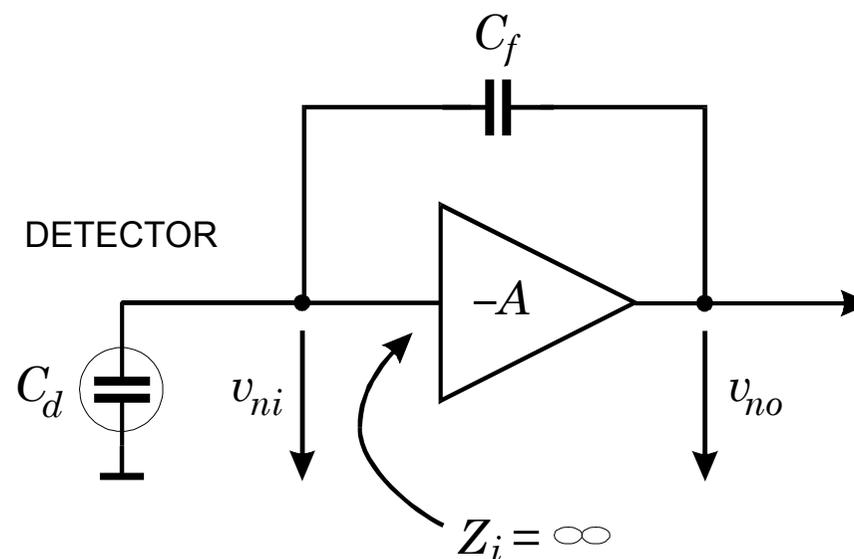
$$Q_{ni} = v_{ni} (C_d + C_f)$$

Signal-to-noise ratio

$$\frac{Q_s}{Q_{ni}} = \frac{Q_s}{v_{ni} (C_d + C_f)} = \frac{1}{C} \frac{Q_s}{v_{ni}}$$

Same result as for voltage amplifier, but here

- the signal is constant and
- the noise grows with increasing  $C$ .



As shown previously, the pulse rise time at the amplifier output also increases with total capacitive input load  $C$ , because of reduced feedback.

In contrast, the rise time of a voltage sensitive amplifier is not affected by the input capacitance, although the equivalent noise charge increases with  $C$  just as for the charge-sensitive amplifier.

## Conclusion

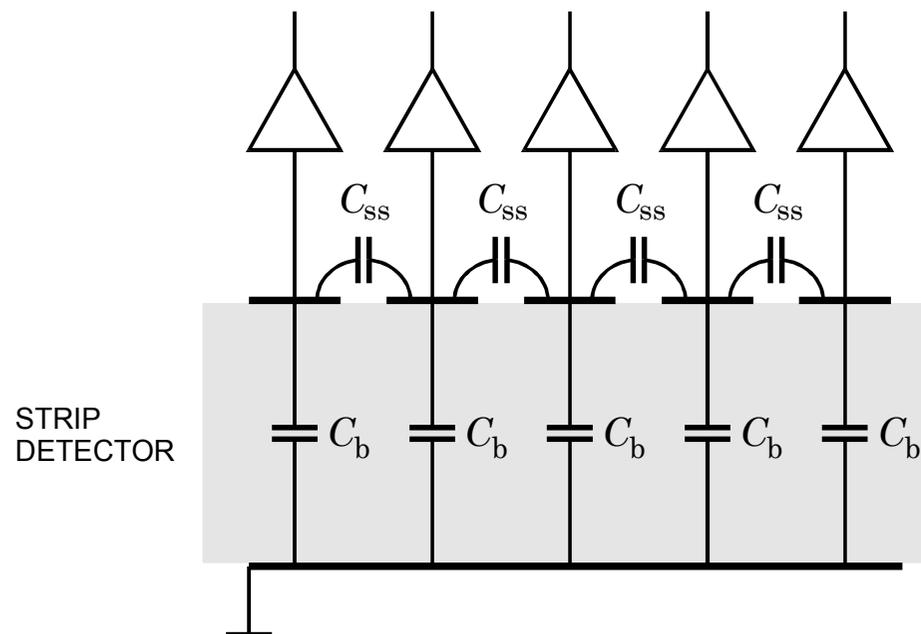
In general

- optimum  $S/N$  is independent of whether the voltage, current, or charge signal is sensed.
- $S/N$  cannot be *improved* by feedback.

Practical considerations, i.e. type of detector, amplifier technology, can favor one configuration over the other.

## 4. Complex Sensors

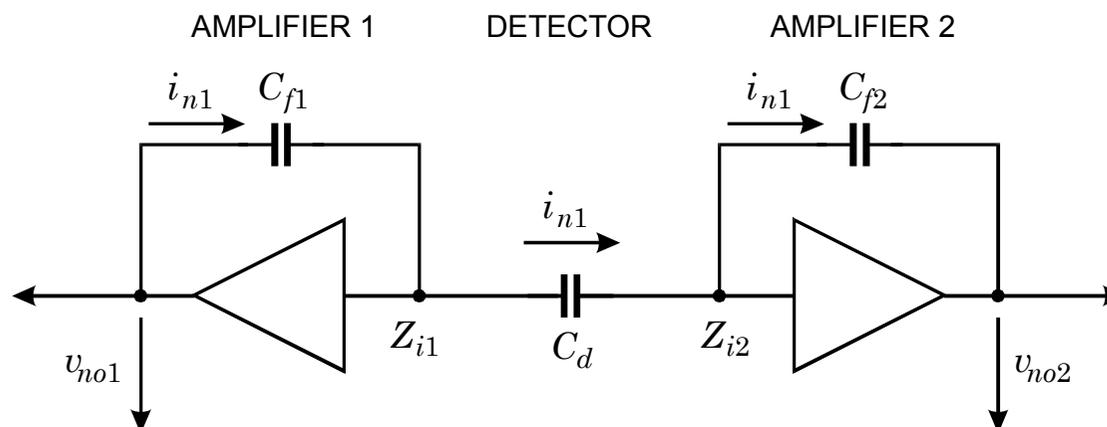
### Cross-coupled noise



Noise at the input of an amplifier is cross-coupled to its neighbors.

## Principle of Noise Cross-Coupling

Consider a capacitance connecting two amplifier inputs, e.g. amplifiers at opposite electrodes of a detector with capacitance  $C_d$ :



First, assume that amplifier 2 is noiseless.

The noise voltage  $v_{no1}$  causes a current flow  $i_{n1}$  to flow through the feedback capacitance  $C_{f1}$  and the detector capacitance  $C_d$  into the input of amplifier 2.

Note that for a signal originating at the output of amplifier 1, its input impedance  $Z_{i1}$  is high ( $\infty$  for an idealized amplifier), so all of current  $i_{n1}$  flows into amplifier 2.

Amplifier 2 presents a low impedance to the noise current  $i_{n1}$ , so its magnitude

$$i_{n1} = \frac{U_{no1}}{X_{C_{f1}} + X_{C_d}} = \frac{U_{no1}}{\frac{1}{\omega C_{f1}} + \frac{1}{\omega C_d}}.$$

The voltage at the output of amplifier is the product of the input current times the feedback impedance,

$$U_{no12} = \frac{i_{n1}}{\omega C_{f2}} = \frac{U_{no1}}{\frac{1}{\omega C_{f1}} + \frac{1}{\omega C_d}} \cdot \frac{1}{\omega C_{f2}} = \frac{U_{no1}}{\frac{C_{f2}}{C_{f1}} + \frac{C_{f2}}{C_d}}.$$

For identical amplifiers  $C_{f1} = C_{f2}$ . Furthermore,  $C_{f2} \ll C_d$ , so the additional noise from amplifier 1 at the output of amplifier 2 is

$$U_{no12} = U_{no1}.$$

This adds in quadrature to the noise of amplifier 2. Since both amplifiers are same,  $U_{no1} = U_{no2}$ , so cross-coupling increases the noise by a factor  $\sqrt{2}$ .

## Cross-Coupling in Strip Detectors

The backplane capacitance  $C_b$  attenuates the signal transferred through the strip-to-strip capacitance  $C_{ss}$ .

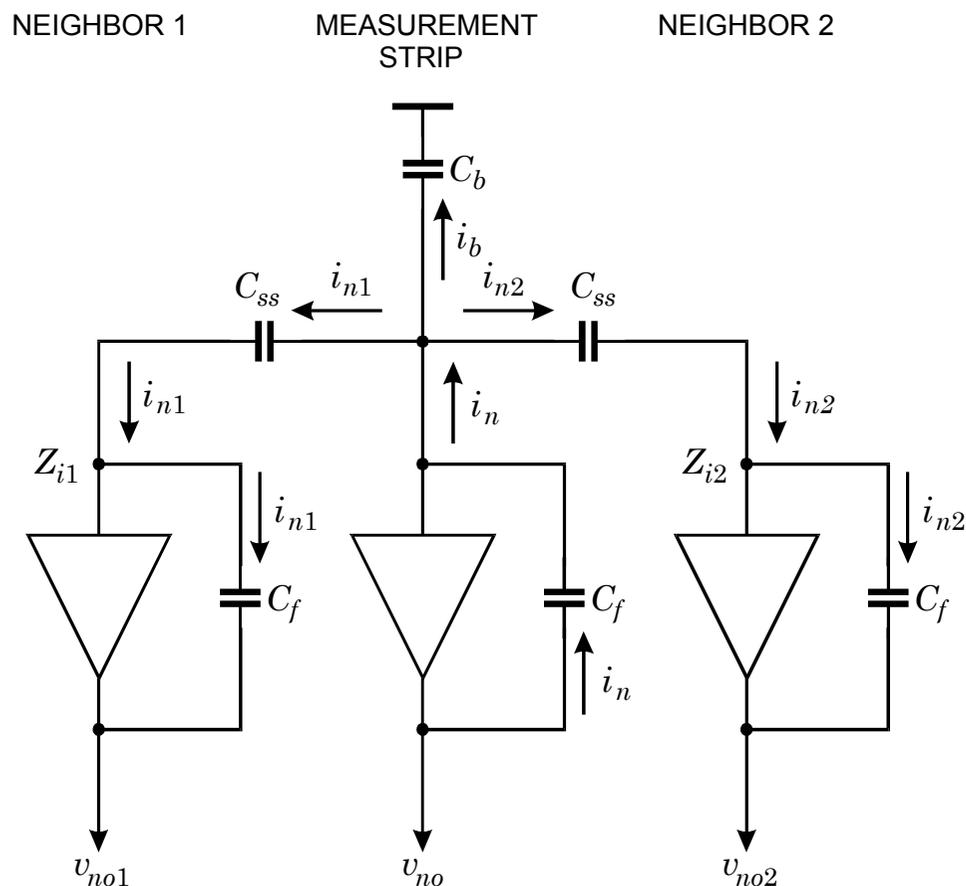
The additional noise introduced into the neighbor channels

$$v_{no1} = v_{no2} \approx \frac{v_{no}}{2} \frac{1}{1 + 2C_b / C_{ss}}$$

For  $C_b = 0$ ,  $v_{no1} = v_{no2} = v_{no} / 2$  and the total noise increases by a factor

$$\sqrt{1 + 0.5^2 + 0.5^2} = 1.22$$

For a backplane capacitance  $C_b = C_{ss} / 10$  the noise increases by 16%.



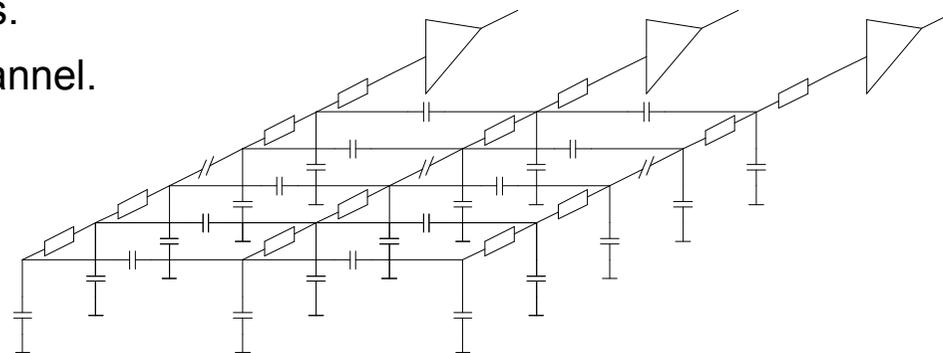
## Strip Detector Model for Noise Simulations

Noise coupled from neighbor channels.

Analyze signal and noise in center channel.

Includes:

- Noise contributions from neighbor channels
- Signal transfer to neighbor channels
- Noise from distributed strip resistance
- Full SPICE model of preamplifier



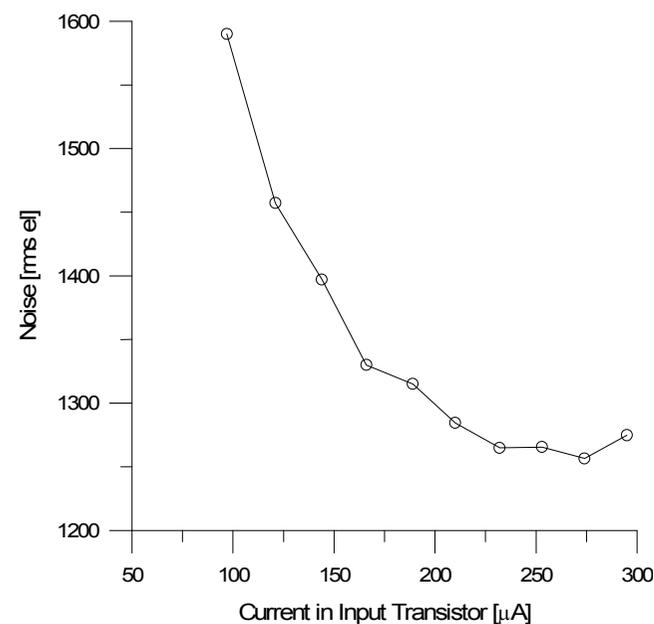
Measured Noise of Module:

p-strips on n-bulk, BJT input transistor

Simulation Results: 1460 el (150  $\mu\text{A}$ )

1230 el (300  $\mu\text{A}$ )

$\Rightarrow$  Noise can be predicted with good accuracy.



## 5. Quantum Noise Limits in Amplifiers

What is the lower limit to electronic noise?

Can it be eliminated altogether, for example by using superconductors and eliminating devices that carry shot noise?

Starting point is the uncertainty relationship

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

Consider a narrow frequency band at frequency  $\omega$ . The energy uncertainty can be given in terms of the uncertainty in the number of signal quanta

$$\Delta E = \hbar \omega \Delta n$$

and the time uncertainty in terms of phase

$$\Delta t = \frac{\Delta \varphi}{\omega},$$

so that

$$\Delta \varphi \Delta n \geq \frac{1}{2}$$

We assume that the distributions in number and phase are Gaussian, so that the equality holds.

Assume a noiseless amplifier with gain  $G$ , so that  $n_1$  quanta at the input yield

$$n_2 = Gn_1$$

quanta at the output.

Furthermore, the phase at the output  $\varphi_2$  is shifted by a constant relative to the input.

Then the output must also obey the relationship  $\Delta\varphi_2\Delta n_2 = \frac{1}{2}$

However, since  $\Delta n_2 = G\Delta n_1$  and  $\Delta\varphi_2 = \Delta\varphi_1$  :

$$\Delta\varphi_1\Delta n_1 = \frac{1}{2G} ,$$

which is smaller than allowed by the uncertainty principle.

This contradiction can only be avoided by assuming that the amplifier introduces noise per unit bandwidth of

$$\frac{dP_{no}}{d\omega} = (G - 1)\hbar\omega ,$$

which, referred to the input, is

$$\frac{dP_{ni}}{d\omega} = \left(1 - \frac{1}{G}\right)\hbar\omega$$

If the noise from the following gain stages is to be small, the gain of the first stage must be large, and then the minimum noise of the amplifier

$$\frac{dP_{ni}}{d\omega} = \hbar\omega$$

At 2 mm wavelength the minimum noise corresponds to about 7K.

This minimum noise limit applies to phase-coherent systems. In systems where the phase information is lost, e.g. bolometers, this limit does not apply.

For a detailed discussion see

C.M. Caves, Phys. Rev. D **26** (1982) 1817-1839

H.A. Haus and J.A. Mullen, Phys. Rev. 128 (1962) 2407-2413